Name: CECS 271 Introduction to Numerical Methods

Exam #2 Take Home 10/28/2020 Due: Friday 30 OCT 2020 (5pm)

Please use this MS Document and fill in the asked for data using MS Word, and when finished save as a pdf document to which please append the published Matlab script file used to generate your answers. Submit this pdf document to the Exam#2 Dropbox on Beachboard by the due date. The Matlab code is the documentation for your answers and should reflect the values that are shown on this submitted exam sheet. Please type in your name in CAPITALS above.

1. Using Matlab with Newton's forward differences approach and this data set, find the value of f(x0) to an accuracy of at least six decimal places. Using stem, plot the data points in red and plot in blue the approximating curve using 1000 points between 0.5 and 2.5. Title the plot ‘Problem 1’ and label the axes. Stem plot the point (x0,f(x0) ) in black on the plot. If the actual value f(x0) = -0.511075, what is the % relative error?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 1 Data Set for Problem 1 %Error= | | | | | | |
| x | 0.6 | 1.25 | Interpolated x0=1.5 | 1.85 | 1.9 | 2.5 |
| f(x) | 0.909690 | -0.990435 |  | 0.384905 | 0.482509 | 0.993593 |
| Coefficients | c(1)= | c(2)= | c(3)= | c(4)= | c(5)= | |

Note: List the coefficients used in the forward differences approach to only 2 decimal places.

2. In 1979, Kentucky Derby winner Spectacular Bid logged a time of 122.4 seconds for the 1.25 mile race. The times at the ¼, ½ and 1 mile poles were 25.4 sec, 49.4 sec and 97.6 sec respectively as shown in this table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 2 Spectacular Bid Pole Times | | | | | | |
| Pole | 0 | ¼ mile | ½ mile | ¾ mile | 1 mile | 1 ¼ miles |
| Time | 0 | 25.4 sec | 49.4 sec |  | 97.6 sec | 122.4 sec |

(a) Use these values to construct a cubic spline for Spectacular Bid's race, and use the spline to predict the time at the ¾

mile pole and find the percent absolute relative error (%ARE) using the actual time of 72.4 sec measured at the ¾ pole.

|  |  |
| --- | --- |
| Quantity | ¾ mile pole |
| Predicted Time |  |
| %ARE |  |

Plot the points of Table #2 along with the cubic spline using 500 points between 0 and 122.4 to give a smooth curve of

the predicted times of Spectacular Bid’s race. Title the plot as ‘Problem 2(a)’, and label the axes.

(b) Use the derivative of the spline to find the horse's speed at the ¼ pole in mph and the speed in mph when crossing the finish line.

|  |  |
| --- | --- |
| Quantity | miles per hour |
| Start Speed |  |
| End Speed |  |

(c) Fit a best-fitting-least-squares cubic of the general form t(x) = a + bx + cx2 + dx3 to Spectacular Bid’s data of Table 2

and list the times (in seconds) it gives at the mile poles of the race in Table 3 below. Display the coefficients a, b, c and d

to 4 decimal places. Plot the data points (use 72.4 sec at the ¾ pole) and the approximating cubic using 250 points over the interval 0 to 1.25 miles. Plot the points and the times given by the cubic equation at the mile poles. Title the plot

‘Problem 2(c)’.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 3 Spectacular Bid Pole Times | | | | | |
| Pole | ¼ | ½ | ¾ | 1 | 1 ¼ |
| BFLS cubic times |  |  |  |  |  |
| Coefficients | a= | b= | c= | d= |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 3. Let A = [ r3 | -5r | 4 | -2 | -1 |  |
| -r | 3 | -1 | r2 | 1 |
| 1 | 2 | 3r | -2 | -1 |
| 4 | -2 | -1 | -2r | 1 |
| 3 | -r | 1 | -1 | -2 | ] |

Using the root finding method of BISECTION, find the value of r=r0 to a tolerance of 1e-6 that will make the determinant of A vanish. Begin the search between values r=-10 and r=10. After finding the value r0, Plot the value of the determinant of A for a thousand values of r between r0-0.5 and r0+0.5, and plot the point (r0, det(A(r0)) in red on the same plot. Title the plot ‘Problem 3’ and include the value of r0 in the title.

Hint: use anonymous function f=@(r) det(A(r))and another for A(r).

ANS: r0=

and f(r0)=

4. Let A=[1,2,-1; 3,-4,2], B=[2,-1; 4,3; -1,5], C=[A; 1,2,3], and D=[A' , B(:,1)]. Find the quantities indicated. Be sure not to forget to provide your Matlab code and its output for documentation of your results listed here.

Ans:

(a) A(:)’ \* B(:)

Ans:

(b) the sum of all the elements in (D\*C – C\*D)\*A’)

Ans:

(c) the sum of the elements in the odd numbered rows of (D’\*C + A\*A’)

Ans: Ans:

(d) the scalar product of row 2 of A and column 3 of C (i.e. the inner product of the two vectors)

(e) the sum of the elements lying on the two main diagonals of F=(B\*B’ –[ C\*A’, B(:,2)])2/100. If an element is at the intersection of both diagonals, include it in both diagonals.

5. Identify whether the set of linear equations has no solution (0), one solution(1), or an infinite number of solutions(Inf).

Ans:

(a) a + b + c =2; 2a-3b+5c=7; -a-2b+c=1

Ans:

(b) a+b-c+d-e=0; a-b-c-d+e=1; -a-b+c-d-e=-1; a-b+c-d+e=1; a+b+c+d+e=0

Ans:

(c) w+4x-2y+5z=0; 3w-x+y-4z=9; -2w+x-2y+3z=-2; w-x+y-2z=2

Ans:

(d) 3x-2y-z=-1; -2x+(4/3)y+(1/3)z=3; x-2y+4z=5;

Ans:

(e) Ax=b where x=[x1,x2,x3], A=[1,1,-1; 1,1,1; 1,-1,1] and b=[3;2;1]